YONSEI MATH COLLOQUIUM

Riemann zeta-function and some conjectures

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Abstract

Assuming the simplicity of the zeros of the Riemann zeta function $\zeta(s)$, Gonek and Hejhal studied the sum

$$J_{-k}(T) := \sum_{0 < \gamma \le T} |\zeta'(\rho)|^{-2k}$$

for real number $k \ge 0$ and conjectured that $J_{-k}(T) \times T(\log T)^{(k-1)^2}$ for any $k \in R$. Assuming Riemann hypothesis and $J_{-1}(T) \ll T$, Ng (NN, PLMS (2004)) proved that the Mertens function $M(x) \ll \sqrt{x}(\log x)^{3/2}$. He pointed out that with the additional hypothesis of $J_{-\frac{1}{2}}(T) \ll T(\log t)^{1/4}$ one gets $M(x) \ll \sqrt{x}(\log x)^{5/4}$.

Here we show that it is possible to obtain $M(x) \ll \sqrt{x}(\log x)^{\alpha}$ for any real number $\alpha \in [5/4, 3/2]$, under similar hypotheses. This is a joint work with Dr. Biswajyoti Saha.

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Talk 17:00 (Tea 16:40) For more details math.yonsei.ac.kr