# Discrete Analysis Seminar 

Alexander Clifton<br>IBS DIMAG

Odd Covers of Graphs

In 1992, Babai and Frankl posed the odd cover problem, asking for the smallest number of complete bipartite graphs for which every edge of $K_{n}$ is contained in an odd number of them. We extend this notion to general graphs, defining an odd cover of $G$ as a collection of complete bipartite graphs where every edge of $G$ is contained in an odd number and every non-edge of $G$ is contained in an even number. Alternatively, the edge set of $G$ can be expressed as the symmetric difference of the edge sets of the complete bipartite graphs in an odd cover. We define $b_{2}(G)$ as the cardinality of the smallest odd cover of $G$.

Over $\mathbb{F}_{2}$, the adjacency matrices of the graphs in any odd cover of $G$ sum to the adjacency matrix of $G$. Using this linear algebra perspective, we determine $b_{2}$ for several families of graphs, such as bipartite graphs and disjoint unions of cycles. We completely determine $b_{2}\left(K_{2 n+1}\right)$ which solves the odd cover problem for odd cliques and prove some partial results for even cliques.

Based on joint work with Calum Buchanan, Eric Culver, Jiaxi Nie, Jason O’Neill, Puck Rombach, and Mei Yin and with Calum Buchanan, Eric Culver, Péter Frankl, Jiaxi Nie, Kenta Ozeki, Puck Rombach, and Mei Yin.

Date: 21th March, 2024
Time: 2:00pm - 3:00pm
Location: 262, Science Building

