Discrete Analysis Seminar

Hyunwoo Lee (이현우) KAIST

Survey on Behrend-type upper bounds for generalised Roth's theorem

Roth's theorem shows that the maximum size of the 3-term arithmetic progression (3-AP) free set in [N] is $O(N/\log \log N)$. The lower bound of the size of the maximum size of 3-AP free set, constructed by Behrend, is at least $N/e^{c\sqrt{\log N}}$ for some positive constant c. In various literature, reducing the gap between Roth's bound and Behrend's bound were extensively studied.

Very recently, Kelley and Meka settled a Behrend-type upper bound for 3-AP free sets. Since finding a 3-AP is equivalent to finding a solution of the equation $x_1 + x_2 = 2y$, a natural extension of Roth's theorem is considering a solution of a similar equation with more variables. Before Kelley and Meka, Schoen and Shkredov showed that there is a Behrend-type upper bound for solution-free sets for specific types of linear equations. In particular, For every $k \ge 5$, if $A \subseteq [N]$ has no nontrivial solution to the equation $\sum_{i \in [k]} x_i = ky$, then $|A| \le O(N/e^{c(\log N)^{c'}})$ for some positive constants c and c'. In this talk, we will discuss Schoen and Shkredov's argument and compare it with Kelley and Meka's argument.

Date: 7th April, 2023 Time: 3:00pm - 4:00pm Location: B103, Science Building



