2022 PDE Workshop

Date January 19th (Wednesday) – 21th (Friday)

Venue Ballroom (3F), Shilla Stay Haeundae, Busan

Organizers

Seick Kim (Yonsei University)

Jongkeun Choi (Pusan National University)

Speakers

Jaewook Ahn (Dongguk University)

Hantaek Bae (UNIST)

Soohyun Bae (Hanbat National University)

Beomjun Choi (POSTECH)

Jongkeun Choi (Pusan National University)

Sukjung Hwang (KIAS)

Jin Woo Jang (POSTECH)

Kyungkeun Kang (Yonsei University)

Seick Kim (Yonsei University)

Bongsuk Kwon (UNIST)

Donghyun Lee (POSTECH)

Mikyoung Lee (Pusan National University)

Taehun Lee (KIAS)

Jinwan Park (Seoul National University)

Jung-Tae Park (KIAS)

Inbo Sim (University of Ulsan)

CMAC 응용해석 및 계산센터 연세대학교 YONSEI UNIVERSITY 부산대학교 PUSAN NATIONAL UNIVERSITY

Schedule

	January 19th	January 20th	January 21th
10:30 - 11:00		Beomjun Choi	Sukjung Hwang
11:05 - 11:35		Hantaek Bae (Online)	Mikyoung Lee
11:40 - 12:10		Jung-Tae Park (Online)	Jin Woo Jang
	Lunch		
14:00 - 14:30	Soohyun Bae	Bongsuk Kwon	
14:35 - 15:05	Jongkeun Choi	Inbo Sim	
	Coffee break		
15:30 - 16:00	Donghyun Lee (Online)	Jinwan Park	
16:05 - 16:35	Jaewook Ahn (Online)	Taehun Lee	
16:40 - 17:10	Kyungkeun Kang	Seick Kim	

January 19th

• 14:00 - 14:30 Soohyun Bae (Hanbat National University)

Title: Existence and uniqueness of radially symmetric singular solutions for supercritical semilinear elliptic equations
Abstract: In this talk, I discuss the existence and uniqueness of radially symmetric singular solutions with supercritical nonlinearity. Asymptotic behavior is the main tool to prove the existence and uniqueness under Kato's condition near 0. I also present an open question on the uniqueness.

• 14:35 - 15:05 Jongkeun Choi (Pusan National University)

Title: Optimal regularity of mixed boundary value problems
Abstract: As is well known, solutions to purely Dirichlet/conormal boundary value problems are smooth when coefficients, data, and boundaries of domains are smooth. However, for mixed Dirichlet-conormal boundary value problems, such a regularity result does not hold near the separation. In this talk, I will present recent results on optimal regularity of the mixed problems for both elliptic and parabolic equations. This talk is based on joint work with Hongjie Dong (Brown) and Zongyuan Li (Rutgers).

• 15:30 - 16:00 Donghyun Lee (POSTECH)

Title: Hölder regularity of the Boltzmann equation past an obstacle
Abstract: In this talk, we consider the Boltzmann equation outside of convex ball with specular reflection boundary condition. Unlike previous results for convex domains, the trajectory which undergoes specular reflection is not differentiable anymore and we prove local $C^{1/2}_{x,v}$ regularity. This is stark difference to other boundary condition (e.g.BV regularity of diffuse BC case). In particular, we introduce novel 'specular singularly' and 'shift method' to measure sharp profile of singularity.

• 16:05 - 16:35 Jaewook Ahn (Dongguk University)

Title: Global solvability and asymptotics of logarithmic chemotaxis systems
Abstract: In this talk, we consider two logarithmic chemotaxis models. As for the first model, a global weak solution that becomes smooth after some waiting time is constructed. This solution stabilizes to a constant steady-state under further assumptions on the domain and the system parameters. Related works on the non-existence of non-constant steady states would be also discussed. As for the second model, a global classical solution is constructed. The possible range of a chemosensitivity parameter is enlarged and a new type of small initial data is found. We discuss long-time asymptotic behaviors of solutions as well.

• 16:40 - 17:10 Kyungkeun Kang (Yonsei University)

Title: Construction of singular solution for the Navier-Stokes equations near boundary in a half-space
Abstract: We study local boundary regularity for the Navier-Stokes equations. We prove that, unlike in the interior case, non-local effects can lead to a violation of local regularity in the spatial variables near the boundary.

January 20th

• 10:30 - 11:00 Beomjun Choi (POSTECH)

Title: Liouville theorem for surfaces translating by sub-affine-critical powers of Gauss curvature
Abstract: We classify the translators to the flows by sub-affine-critical powers of Gauss curvature in $\mathbb{R}^3$. If α denotes the power, this is a Liouville theorem for degenerate Monge-Ampere equations $\det D^2 u = (1+|Du|^2)^{2-\frac{1}{2\alpha}}$ for $0 < \alpha < 1/4$. For the affine-critical-case $\det D^2 u = 1$, the classical result by Jorgens, Calabi and Pogorelov shows the level curves of given solution are homothetic ellipses. In our case, the level curves converge asymptotically to a round circle or a curve with k-fold symmetry for some $k > 2$. More precisely, these curves are closed shrinking curves to the $\frac{\alpha}{1-\alpha}$-curve shortening flow that were previously classified by B. Andrews in 2003. This is a joint work with K. Choi and S. Kim.

• 11:05 - 11:35 Hantaek Bae (UNIST)

Title: Introduction to active models and viscoelastic fluids
Abstract: In this talk, we first introduce continuum models of self-propelled particles in Newtonian fluids. We then proceed to extend these models in viscoelastic fluids. As a priori study on these extended models, we also introduce viscoelastic fluids as well. These are joint work with Woojae Lee and Jaeyong Shin.

• 11:40 - 12:10 Jung-Tae Park (KIAS)

Title: Marcinkiewicz regularity for singular parabolic p-Laplace type equations with measure data
Abstract: In this talk, we consider a parabolic p-Laplace type equation when the right-hand side is a signed Radon measure with finite total mass, whose model is

$$u_t - \operatorname{div}\left(|Du|^{p-2}Du\right) = \mu \quad \text{in } \Omega \times (0,T) \subset \mathbb{R}^n \times \mathbb{R}.$$

In the singular range $\frac{2n}{n+1} < p \leq 2 - \frac{1}{n+1}$, we discuss integrability results for the spatial gradient of a solution in the Marcinkiewicz space, under a suitable density condition of the right-hand side measure μ.

• 14:00 - 14:30 Bongsuk Kwon (UNIST)

Title: Plasma solitary waves

Abstract: We study the asymptotic linear stability of a two-parameter family of solitary waves for the isothermal Euler–Poisson system. When the linearized equations about the solitary waves are considered, the associated eigenvalue problem in L^2 space has a zero eigenvalue embedded in the neutral spectrum, i.e., there is no spectral gap. To resolve this issue, use is made of an exponentially weighted L^2 norm so that the essential spectrum is strictly shifted into the left-half plane, and this is closely related to the fact that solitary waves exist in the super-ion-sonic regime. Furthermore, in a certain long-wavelength scaling, we show that the Evans function for the Euler–Poisson system converges to that for the Korteweg–de Vries (KdV) equation as an amplitude parameter tends to zero, from which we deduce that the origin is the only eigenvalue on its natural domain with algebraic multiplicity two. We also show that the solitary waves are spectrally stable in L^2 space. Moreover, we discuss (in)stability of large amplitude solitary waves.

• 14:35 - 15:05 Inbo Sim (University of Ulsan)

Title: New case study of existence results for a class of (p, q)-Laplace problems

Abstract: In this talk, we investigate the existence of solutions to a class of (p, q)-Laplace problems. More precisely, first, we study the existence of a nontrivial nonnegative solution to (p, q)-Laplace equations involving two nonlinear terms, one grows as s with $q < s < p$ and the other possibly has critical growth. Our argument is based on the concentration–compactness principle by P.L. Lions and the Ekeland variational principle. Second, we show various existence results of positive solutions to the one-dimensional generalized double phase problems including the existence of at least two or three positive solutions according to the behaviors of a nonlinearity near zero and infinity via the Krasnoselskii type fixed point theorem.

• 15:30 - 16:00 Jinwan Park (Seoul National University)

Title: The regularity theory for the double obstacle problems
Abstract: In this talk, I will introduce the regularity of the free boundary for the double obstacle problems.

First, I am going to introduce the proof of local C^1 regularity of free boundaries for the elliptic double obstacle problem with an upper obstacle ψ,

$$\Delta u = f\chi_{\Omega(u)\cap\{u<\psi\}} + \Delta\psi\chi_{\Omega(u)\cap\{u=\psi\}}, \qquad u \le \psi \quad \text{in } B_1,$$

where $\Omega(u) = B_1 \setminus (\{u = 0\} \cap \{\nabla u = 0\})$ under a thickness assumption for u and ψ. The function ψ satisfies

$$\psi \in C^{1,1}(B_1) \cap C^{2,1}(\overline{\Omega(\psi)}), \quad \Omega(\psi) = B_1 \setminus (\{\psi = 0\} \cap \{\nabla\psi = 0\}).$$

This is a joint work with Ki-ahm Lee and Henrik Shahgholian.

Next, I will talk about the parabolic problem. This is a joint work with Ki-ahm Lee.

• 16:05 - 16:35 Taehun Lee (KIAS)

Title: Gauss curvature flow with an obstacle
Abstract: The Gauss curvature flow describes the changing shape of tumbling stones by water waves. This flow can be formulated as a parabolic Monge-Ampère type equation which can be in general degenerate. As in most other flows, singularities are developed under the flow if the initial data is closed hypersurface. In this talk, we consider one of the natural free boundary problems in the flow, *the Gauss curvature flow with an obstacle*, and discuss its regularity and final shape that is prevented from the development of singularities. This work is joint with Ki-Ahm Lee.

• 16:40 - 17:10 Seick Kim (Yonsei University)

Title: Gaussian estimates for second order parabolic equations in non-divergence form
Abstract: We construct the fundamental solution of second order parabolic equations in non-divergence form under the assumption that the coefficients are of Dini mean oscillation in the spatial variables. We also prove that the fundamental solution satisfies a sub-Gaussian estimate. In the case when the coefficients are Dini continuous in the spatial variables and measurable in the time variable, we establish the Gaussian bounds for the fundamental solutions. We present a method that works equally for second order parabolic systems in non-divergence form.

January 21th

• 10:30 - 11:00 Sukjung Hwang (KIAS)

Title: Existence of weak solutions of porous medium equation with a drift
Abstract: In this talk, we consider porous medium type equations with the divergence form of drift that can be applied to fluid dynamics and math biology. We explain the existence of nonnegative weak solutions in the Wasserstein space where the nonlinear diffusion and initial data affect the scaling invariant classes of the drift. Moreover, we discuss uniqueness result and application for a repulsive Keller-Segel model. This is joint work with K. Kang (Yonsei Univ.) and H. Kim (Hannam Univ.).

• 11:05 - 11:35 Mikyoung Lee (Pusan National University)

Title: Hölder regularity results for a class of obstacle problems with nonstandard growth conditions
Abstract: In this talk we discuss about Hölder regularity for the gradient of local solutions to obstacle problems with nonstandard growth. These results cover standard, variable exponent, double phase and Orlicz growth cases. The talk is based on joint work with Arttu Karppinen (University of Turku, Finland).

• 11:40 - 12:10 Jin Woo Jang (POSTECH)

Title: LTE and Non-LTE Solutions in Gases Interacting with Radiation
Abstract: The goal of this talk is to discuss a class of kinetic equations describing radiative transfer in gases which include also the interaction of gas molecules with themselves. We first introduce a system of kinetic PDEs that describes the dynamics of gas molecules coupled with an equation for photons' radiative transfer. We then discuss several scaling limits and introduce some Euler-like systems coupled with radiation as an aftermath of specific scaling limits. We consider scaling limits in which local thermal equilibrium (LTE) holds, as well as situations in which this assumption fails (non-LTE). We observe that the structure of the equations describing the gas-radiation system is very different in the LTE and non-LTE cases. We prove the existence of stationary solutions to the resulting limit models in the LTE case. Lastly, we will also prove the non-existence of stationary solutions with zero velocities in a non-LTE situation. This is a joint work with Juan J. L. Velazquez at Bonn.