

Scale invariant regularity estimates for second order elliptic equations with lower order coefficients in optimal spaces

Dr. Georgios Sakellaris

2021.12.14 (Tue) 4:00pm (ZOOM)

Abstract : We show local and global scale invariant regularity estimates for subsolutions and supersolutions to the equation $-\operatorname{div}(A\nabla u + bu) + c\nabla u + du = -\operatorname{div} f + g$, assuming that A is elliptic and bounded. In the setting of Lorentz spaces, under the assumptions $b, f \in L_{n,1}$, $d, g \in L_{n/2,1}$ and $c \in L_{n,q}$ for $q \leq \infty$, we show that, with the surprising exception of the reverse Moser estimate, scale invariant estimates with "good" constants (that is, depending only on the norms of the coefficients) do not hold in general. On the other hand, assuming a necessary smallness condition on b, d or c, d , we show a maximum principle and Moser's estimate for subsolutions with "good" constants. We also show the reverse Moser estimate for nonnegative supersolutions with "good" constants, under no smallness assumptions when $q < \infty$, leading to the Harnack inequality for nonnegative solutions and local continuity of solutions. Finally, we show that, in the setting of Lorentz spaces, our assumptions are the sharp ones to guarantee these estimates.

