# The 3rd Meeting of Young Researchers in PDEs

**Dates**: Dec. 13 – 15, 2018

**Venue**: Room 8101 KIAS, Seoul, Korea

**Host**: CMAC (Center for Mathematical Analysis & Computation, Yonsei University)

**Sponsors**: Institute for Mathematical Science (Yonsei University) & CMAC & KIAS

## Speakers

Ken Abe (Osaka City University)

Jongkeun Choi (KIAS)

Jaywan Chung (Korea Electrotechnology Research Institute)

Sukjung Hwang (Yonsei University)

Norihisa Ikoma (Keio University)

Jin Woo Jang (IBS)

Yunsoo Jang (Yonsei University)

In-Jee Jeong (KIAS)

Soyeun Jung (Kongju National University)

Lami Kim (Yonsei University)

Soojung Kim (KIAS)

Donghyun Lee (Postech)

Jaeseung Lee (Seoul National University)

Jinkai Li (South China Normal University)

Tongseok Lim (Shanghaitech University)

Byungsoo Moon (Incheon National University)

Sung-Jin Oh (KIAS)

Takahiro Okabe (Osaka University)

Jung-Tae Park (KIAS)

Yong Yu (Chinese University of Hong Kong)

## Organizers

Hantaek Bae (UNIST)

Moon-Jin Kang (Sookmyung Women's University)

In-Jee Jeong (KIAS)

Sung-Jin Oh (KIAS)

Jinmyoung Seok (Kyonggi University)

Minsuk Yang (Yonsei University)

# Time Table

| | Dec. 13th | Dec. 14th | Dec. 15th |
|---|---|---|---|
| 10:00~10:30 | | Norihisa Ikoma | Tongseok Lim |
| 10:30 ~11:00 | | Donghyun Lee | Soojung Kim |
| 11:00~11:15 | | Break | Break |
| 11:15~11:45 | | Jinkai Li | Takahiro Okabe |
| 11:45~12:15 | | Jaeseung Lee | Sukjung Hwang |
| 12:10~14:00 | Opening | Lunch | |
| 14:00~14:30 | Ken Abe | Yunsoo Jang | |
| 14:30~15:00 | Lami Kim | Jin Woo Jang | |
| 15:00~15:30 | Break | Break and poster session | |
| 15:30~16:00 | Yong Yu | Byungsoo Moon | |
| 16:00~16:30 | Jung-Tae Park | Soyeun Jung | |
| 16:30~16:45 | Break | Break | |
| 16:45~17:15 | Sung-Jin Oh | Jaywan Chung | |
| 17:15~17:45 | In-Jee Jeong | Jongkeun Choi | |

# Dec. 13th

## Ken Abe (Osaka City University)

**Title**: Vanishing viscosity limits for axisymmetric flows with boundary

**Abstract**: In this talk, I will explain some result on existence of global weak solutions to the Euler equations in an infinite cylinder for axisymmetric initial data without swirl. I first explain a global well-posedness result for the Navier-Stokes equations for axisymmetric data without swirl in $L^p$, $3 \leq p < \infty$, subject to the Neumann boundary condition. We then apply a vanishing viscosity method and construct global weak solutions to the Euler equations. This talk is based on arXiv:1806.04811.

## Lami Kim (Yonsei University)

**Title**: On the mean curvature flow of grain boundaries.

**Abstract**: Suppose that $\Gamma_0 \subset \mathbb{R}^{n+1}$ is a closed countably $n$-rectifiable set whose complement $\mathbb{R}^{n+1} \setminus \Gamma_0$ consists of more than one connected component. Assume that the $n$-dimensional Hausdorff measure of $\Gamma_0$ is finite or grows at most exponentially near infinity. Under these assumptions, we prove a global-in-time existence of mean curvature flow in the sense of Brakke starting from $\Gamma_0$. There exists a finite family of open sets which move continuously with respect to the Lebesgue measure, and whose boundaries coincide with the space-time support of the mean curvature flow. This is a joint work with Yoshihiro Tonegawa.

## Yong Yu (Chinese University of Hong Kong)

**Title**: Debye layer limit in charge conserving Poisson-Boltzmann equation.

**Abstract**: The Poisson-Boltzmann equation is a useful equation to understand physiological interfaces, polymer science, electron interactions in a semiconductor, or more. The charge conserving Poisson-Boltzmann equation is a non-local version of the standard Poisson-Boltzmann equation, which provides us with equilibrium solutions to Poisson-Nernst-Planck equation under no flux boundary condition. In this talk we will quantitatively discuss the so-called Debye layer phenomenon in charge conserving Poisson-Boltzmann equation. Under the neutrality assumption, we provide a novel and explicit formula to evaluate the limiting electric potential inside the physical domain as a small parameter approaching to zero. In fact due to the non-local term in charge conserving Poisson-Boltzmann equation, the limiting value of electric potential inside the physical domain is not a-priorily known. Our formula reveals the crucial relationships between the limiting electric potential inside the physical domain and the Dirichlet data of the electric potential prescribed on the physical boundary. This is a joint work with Chia-Yu Hsieh.

## Jung-Tae Park (KIAS)

**Title**: Global regularity for quasilinear parabolic equations involving measure data.

**Abstract**: In this talk we present global Calderón-Zygmund type estimates for parabolic $p$-Laplace type problems in bounded nonsmooth domains when the right-hand side is a signed Radon measure with finite mass. We define proper solutions and provide minimal conditions which guarantee regularity results for such measure data problems. This is joint work with Sun-Sig Byun and Pilsoo Shin.

## Sung-Jin Oh (KIAS)

**Title**: On the Cauchy problem for the Hall MHD equations without resistivity.

**Abstract**: In this talk, I will describe a recent work with I.-J. Jeong on the Cauchy problem for the Hall MHD equation without resistivity. This PDE, first investigated by Lighthill, is a one-fluid description of magnetized plasma with a quadratic second-order correction term (Hall current term), which takes into account the motion of electrons relative to positive ions. We demonstrate both ill and wellposedness of the Cauchy problem depending on the initial data. In particular, the focus of this talk will be to describe conditions on the initial data that ensure wellposedness.

## In-Jee Jeong (KIAS)

**Title**: Ill-posedness of the Hall-MHD equations without resistivity.

**Abstract**: We show that the incompressible Hall-MHD equations without magnetic resistivity is ill-posed in Sobolev spaces in the following strong sense: for all $H^s$ with $s > 11$, either there is non-existence or non-uniqueness in the class $L^\infty([0,T); H^s)$, or there is $H^s$ norm-inflation near the trivial solution. In the course of proving this nonlinear result, we first consider the linearized problem near a class of special stationary magnetic fields, which we show to be ill-posed in Sobolev spaces higher than the energy space by approximately solving the system up to a few leading orders in frequency. Then this approximation scheme can be adjusted to handle the nonlinear case, which demonstrates robustness of the method. This is joint work with Sung-Jin Oh.

# Dec. 14th

## Norihisa Ikoma (Keio University)

**Title**: Existence of infinitely many solutions to fractional scalar field equations.

**Abstract**: This talk is concerned with the existence of infinitely many solutions to fractional scalar field equations. We mainly consider the fractional Laplacian with a general nonlinearity and the nonlinearity is assumed to be so-called zero mass. The aim of this talk is to establish a fractional counter part of Struwe (1982) and Berestycki–Lions (1983). If time permits, some related results for $(-\Delta + 1)^s$ with $0 < s < 1$ are also presented.

## Donghyun Lee (Postech)

**Title**: The Boltzmann equation in bounded domains.

**Abstract**: Dynamics of many particle system can be described by PDE of probability density function. The Boltzmann equation in kinetic theory is one of the most famous equation which describes rarefied gas dynamics. One of main property of the Boltzmann is decaying to equilibrium as time goes to infinity. This equation and its asymptotics has been studied for several decades ago, but boundary problem of the Boltzmann equation has been widely open because of singular behavior of boundary Boltzmann problem. We introduce new framework to study pointwise behavior of the equation in general boundary problems.

## Jinkai Li (South China Normal University)

**Title**: Entropy-bounded solutions of the compressible Navier-Stokes equations with far field vacuum.

**Abstract**: The entropy is one of the fundamental states of a fluid and, in the viscous case, the equation that it satisfies is highly singular in the region close to the vacuum. In spite of its importance in the gas dynamics, the mathematical analyses on the behavior of the entropy near the vacuum region, were rarely carried out; in particular, in the presence of vacuum, either at the far field or at some isolated interior points, it was unknown if the entropy remains its boundedness. We will show in this talk that the ideal gases retain their uniform boundedness of the entropy, locally or globally in time, if the vacuum occurs at the far field only and the density decays slowly enough at the far field. Precisely, we consider the Cauchy problem to the one-dimensional full compressible Navier-Stokes equations, and establish the local and global existence and uniqueness of entropy-bounded solutions, in the presence of vacuum at the far field only. It is also shown that, different from the case that with compactly supported initial density, the compressible Navier-Stokes equations, with slowly decaying initial density, can propagate the regularities in the inhomogeneous Sobolev spaces. These are joint works with Zhouping Xin.

## Jaeseung Lee (Seoul National University)

**Title**: Emergent dynamics of Lohe-type self alignment models.

**Abstract**: Collective coherent motions are ubiquitous in classical and quantum many-body systems. We study the particle swarming model on the unit sphere and orientation flocking model which can be derived from the Lohe matrix model. In this model, the particles are governed by the first order ODE and their Euclidean norm is preserved. We give a rigorous derivation of PDE and study the stability/instability properties of the model.

## Yunsoo Jang (Yonsei University)

**Title**: Global gradient estimates from composite materials.

**Abstract**: In this talk, we study global gradient estimates for weak solutions from composite materials. We assume that the domain is composed of a finite number of disjoint subdomains with Reifenberg flat boundaries and the coefficients are merely measurable in one variable and have small BMO semi-norms in the other variables on each subdomain. Our proof is based on a new observation for disjoint Reifenberg flat domains $\Omega^k$ and $\Omega^l$ that the normal vectors at $P \in \partial\Omega^k$ and $Q \in \partial\Omega^l$ are almost opposite if $P$ and $Q$ are close enough.

## Jin Woo Jang (IBS)

**Title**: Gain of regularity for the relativistic collision operator.

**Abstract**: In this talk we study a regularity property for the gain part of the relativistic Boltzmann collision operator when the collisional cross-section covers the full-range of generic hard and soft potentials with angular cut-off. The aim of this talk is to present two different proofs based on the Fourier transform.

## Byungsoo Moon (Incheon National University)

**Title**: On the wave-breaking phenomena and global existence for the periodic rotation-two-component Camassa-Holm system.

**Abstract**: In this talk, we consider the periodic rotation-two-component Camassa-Holm system, which can be derived from the f-plane governing equations for the geophysical water waves with a constant underlying current. The nonlocal nonlinearities on blow-up criteria and wave-breaking phenomena are established. Finally, a sufficient condition for global solution is obtained by using a method of the Lyapunov function.

## Soyeun Jung (Kongju National University)

**Title**: Turing patterns in parabolic systems of conservation laws and numerically observed stability of periodic waves.

**Abstract**: Turing patterns on unbounded domains have been widely studied in systems of reaction-diffusion equations. However, up to now, they have not been studied for systems of conservation laws. Here, we derive conditions for Turing instability in conservation laws and use these conditions to find families of periodic solutions bifurcating from uniform states, numerically continuing these families into the large-amplitude regime.

## Jaywan Chung (Korea Electrotechnology Research Institute)

**Title**: Maximum efficiency problem in thermoelectricity.

**Abstract**: The thermoelectric effect is the direct conversion between temperature difference and electrical energy. A thermoelectric generator using the effect allows waste heat recovery but a hindrance to the utilization of the generator is its low efficiency. In this talk, I will introduce a PDE describing a thermoelectric generator and some observations to improve its efficiency by choosing appropriate coefficients of the PDE. Since the coefficients correspond to thermoelectric material properties, the observations suggest guidelines for better thermoelectric materials.

## Jongkeun Choi (KIAS)

**Title**: Optimal regularity of mixed problems.

**Abstract**: I will present the unique $W^{1,p}$ solvability of second order divergence form elliptic equations with mixed Dirichlet-conormal boundary conditions, with $p$ being in the optimal range $(4/3, 4)$. This is joint work with Hongjie Dong and Zongyuan Li.

# Dec. 15th

## Tongseok Lim (Shanghaitech University)

**Title**: Optimal transport and martingale optimal transport, and their applications in economics and finance.

**Abstract**: The theory of Optimal Transport (OT) has developed rapidly during the last 30 years, partly because OT has constantly found applications in many areas, not only in mathematics, but also in economics and other social sciences. More recently a variant of OT, called Martingale Optimal Transport (MOT), has been introduced and extensively studied by mathematical finance community who observed the close connection of MOT with the model-independent robust option pricing and hedging. In this course we give a brief introduction of OT and MOT, from basic economic and financial motivations to some mathematical theory and open problems.

## Soojung Kim (KIAS)

**Title**: Nematic liquid crystal flows in applied magnetic fields.

**Abstract**: In this talk, we consider global solutions to the Ericksen-Leslie system modeling the hydrodynamic flow of nematic liquid crystals. In the presence of an applied magnetic field, the orientation of liquid crystals can be easily aligned along the direction of the external field. We discuss dynamical instabilities of global solutions to the Ericksen-Leslie system due to magnetic fields in dimension two.

## Takahiro Okabe (Osaka University)

**Title**: Remark on the strong solvability of the Navier-Stokes equations in the weak $L^n$ space.

**Abstract**: We consider the incompressible Navier-Stokes equation on the whole space $\mathbb{R}^n$, $n \geq 3$. Especially, we consider the Cauchy problem in the frame work of the weak Lebesgue space $L^{n,\infty}(\mathbb{R}^n)$ with scale invariant forces in $BC([0,\infty); L^{\frac{n}{3},\infty}(\mathbb{R}^n))$. It is well known that $C_0^\infty(\mathbb{R}^n)$ is not dense in $L^{n,\infty}(\mathbb{R}^n)$ and that the Stokes semigroup is not strongly continuous at $t = 0$. For this difficulty, the existence of local (mild) solutions and its uniqueness is not yet completely clarified in $L^{n,\infty}(\mathbb{R}^n)$. Firstly we consider the local existence of weak mild solutions of (N-S) with restriction the singularity of initial data and external forces. Due to the local solution we may apply to the uniqueness theorem within weak mild solutions in the class $BC([0,\infty); L^{n,\infty}(\mathbb{R}^n))$. Finally, we clarified the necessary condition for initial data which causes the weak mild solution in $BC([0,\infty); L^{n,\infty}(\mathbb{R}^n))$.

**References**

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## Sukjung Hwang (Yonsei University)

**Title**: Hölder continuity of nonlinear PDEs

**Abstract**: We discuss the Hölder continuity of porous medium type equations with drift terms. In other words, a proper $L^p$ type class of drift vector fields to achieve Hölder continuity of a weak solution is the main ingredient of this talk. As an application, the results may imply the same regularity of porous medium type Keller-Segel systems. In addition, we discuss corresponding results for fast diffusion type equations.